TRINITY COLLEGE

Mathematics Methods Units 3 & 4
Test 5 2016

Calculator Assumed

Normal Distribution, Sampling & Sample Proportions

STUDENT'S NAME MARKING

DATE: Thursday 25th August **TIME:** 50 mins **MARKS**: 46

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser.
Special Items: Up to three Calculators, 1 side A4 page of notes

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine, if any, the source of sampling bias in the following sampling procedures.

(a) To predict the outcome of a referendum on daylight saving a public opinion poll telephones people randomly selected from the telephone directory. [2]

- Not everyone is in the telephone directory.
- Any reasonable answer involving selection bias.

(b) To survey parental attitude toward a uniform change at a school the deputy principal selected at random 100 students from the school roll and sent out questionnaires to their parents. [2]

- possible response bias
- parents who have multiple students at the school are more likely to be chosen.
- Any reasonable answer.

2. (13 marks)

The Tasty Crumpet Factory produces crumpets whose thickness is normally distributed with a mean thickness of 24 mm and a standard deviation of 1.6 mm.
Consumer research by management indicates that crumpets more than 28 mm thick will not fit into a standard automatic toaster and those with thickness less than 21 mm are unsuitable for toasting as they dry out too quickly and are ejected burnt.

(a) Determine the percentage of crumpets made at the factory which are unsuitable for toasting. [3]

$X \sim N(24, 1.6^2)$

$P(21 \leq X \leq 28) = 1 - 0.9634$

$= 0.0366.$

$= 3.66\%$

(b) A crumpet chosen at random is found to be not suitable for toasting. What is the probability that it is too thin? [2]

$P(X \leq 21 \mid \text{Not suitable}) = \dfrac{P(X \leq 21)}{0.0366}$

$= \dfrac{0.0304}{0.0366}$

$= 0.8303.$

(c) Calculate the thickness exceeded by a quarter of the crumpets produced on a particular day. [2]

$P(X > k.) = 0.25$

$k =$

25.08 mm

(d) Determine the probability that the thickness of a crumpet produced on any given day is exactly 23 mm. [1]

0

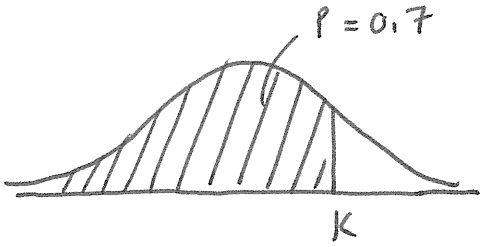
(e) A sample of 200 crumpets were taken. Determine the probability that no more than 10 of these crumpets were unsuitable for toasting. [3]

Y : # crumpets from 200 not suitable.

$Y \sim B(200, 0.0366)$

$P(Y \leq 10) = 0.8813$

(f) Determine the 0.7 quantile and give a worded statement explaining what it means. [2]



$P(X \leq K) = 0.7$

$K = 24.84$ mm

70% of the crumpets have a width less than 24.84 mm.

3. (7 marks)

The percentage marks of students sitting a national mathematics test were found to be normally distributed. Certificates of Credit were awarded to 12.12% of students that sat the test and achieved a mark between 75% and 80%.
15.87% of all students scored at least 80% and were awarded Certificates of Distinction.

(a) Show the mean score for the test was 68% and the standard deviation was 12%. [4]

$P(X \geq 75) = 0.1212 + 0.1587$ $\quad z = 0.5831$

$= 0.2799$

$P(X \geq 80) = 0.1587$ $\quad z = 1$

$$0.5831 = \frac{75 - \mu}{\sigma}$$

$$1 = \frac{80 - \mu}{\sigma}$$

$\therefore \quad \mu = 68\%$.

$\sigma = 12\%$.

(b) A Certificate of Proficiency was awarded to a student achieving a mark no lower than 0.25 standard deviations below the mean and no higher than a Credit score. At a particular school, 200 students sat the test.
How many students would receive Certificates of Proficiency? [3]

$K = 68 - 0.25(12)$

$= 65$

$P(65 \leq X \leq 75) = 0.3188$

$0.3188 \times 200 = 63.76$

$\sim$ 64 students.

4. (4 marks)

Flour from a mill is poured into "1 kg" bags for sale to the public. In fact, the weight of flour in the bags is normally distributed with a mean of 1005 g and a standard deviation of 4 g.

(a) What is the probability that a randomly selected bag of flour is under the marked weight? [2]

$P(X < 1000) = 0.1056$

(b) The mill owners decide that no more than 1% of the bags should be underweight. They increase the mean weight of the bags without changing the standard deviation.
What should be the mean weight of the bags (to the nearest 0.1 g) to ensure that no more than 1% of bags are underweight? [2]

$\sigma = 4$

From standard Normal Dist.

$K = -2.326$

$$-2.326 = \frac{1000 - \mu}{4}$$

$$\mu = 1009.3$$

5. (4 marks)

From a population of people who shop at a particular game store it is known the proportion of gamers who prefer a PlayStation is 54%.

(a) Of a random sample of 300 gamers, 174 indicated they preferred a PlayStation. What is the value of the sample proportion $\hat{p}$? [1]

$$\hat{p} = \frac{174}{300}$$

$$= 0.58$$

(b) Determine the approximate probability that, in a random sample of 300 gamers, the proportion who prefer a PlayStation is greater than or equal to the value of $\hat{p}$ observed in this particular sample. [3]

$$\mu = 0.54$$

$$\sigma = \sqrt{\frac{0.54(1-0.54)}{300}}$$

$$= 0.0288$$

$$\hat{P} \sim N(0.54, 0.0288^2)$$

$$P(\hat{P} \geq 0.58) = 0.0824$$

6. (11 marks)

It is known that at a certain school, the proportion of students who play Pokémon Go is ρ.
A random sample of 150 students was selected and of these students 112 played Pokémon Go.

(a) Briefly describe a suitable method to achieve a random sample of 150 students from the school. [2]

Assign a number $1 \to n$ to each student then randomly generate a sample of 150.

(b) Determine a point estimate for the proportion of students who play Pokémon Go. [1]

$\hat{p} = \frac{112}{150}$ $\quad \hat{p} = 0.75$

(c) Determine a 90% confidence interval for ρ. [3]

$z = 1.645$

$\sigma = \sqrt{\frac{0.75(0.25)}{150}}$

$= 0.0354$

90% C.I

$0.688 \leq \hat{p} \leq 0.8057$

(d) A second sample of 150 students was taken. In this second sample the number of students found to be playing Pokémon Go was 101. Use the confidence interval from (c) to determine if the students in the second sample are statistically different to those of the first sample. [2]

$\hat{p} = \frac{101}{150}$

$= 0.673$

0.673 does not lie within the 90% C.I for $\hat{p}$

$\therefore$ statistically different.

(e) In a third sample of 150 students, the 99% confidence interval for ρ was $0.618 \leq \rho \leq 0.808$. How many students in the sample do you expect to play Pokémon Go? [2]

$\hat{p} = \frac{0.618 + 0.808}{2}$

$= 0.713$

$n = 107$ students.

or

$92.7 \leq n \leq 121.2$

$92 \leq n \leq 122$

(f) Comment on the width of the confidence intervals in (c) and (e). [1]

width of 99% C.I is larger than that of a 90% C.I.

7. (3 marks)

The proportion of students in a state that have Facebook accounts is ρ. A random sample of n students was selected and 50 indicated that they do not have Facebook accounts.
Determine n if the magnitude for the margin of error for the 99% confidence interval for ρ is 0.05.

$$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.05 = 2.576\sqrt{\frac{\frac{n-50}{n}\left(1-\frac{n-50}{n}\right)}{n}}$$

$$n = 51,336.$$